



首都师范大学数学科学学院

School of Mathematical Sciences Capital Normal University

# The 10<sup>th</sup> East Asian Number Theory Conference

*February 13-17, 2023*

**Capital Normal University**

**(Beijing, China)**

## Scientific organizers

**Sunghan Bae** (KAIST, Daejeon)

**Liang-Chung Hsia** (NTNU, Taipei)

**Yuichiro Taguchi** (TITech, Tokyo)

**Fei Xu** (CNU, Beijing)

## Local organizers

**Shun Tang** (CNU, Beijing)

**Jilong Tong** (CNU, Beijing)

**Fei Xu** (CNU, Beijing)

## Online Meeting Information

**Zoom Meeting ID: 818 3880 4550**

**Passcord: 490295**

**<https://us06web.zoom.us/j/81838804550?pwd=MEZPQUxvTjFpZG0wcGZyT3lYaIRqZz09>**

## Sponsors

© National Natural Science Foundation of China

© School of Mathematical Sciences, Capital Normal University

© Academy for Multidisciplinary Studies, Capital Normal University

© Beijing National Center for Applied Mathematics

The following time table is Beijing time.

Taipei time is the same as Beijing time.

Tokyo time and Seoul time is one hour later than Beijing time.

Conference Schedule					
Date/Time	Feb.13 (Monday)	Feb.14 (Tuesday)	Feb.15 (Wednesday)	Feb.16 (Thursday)	Feb.17 (Friday)
9:00-10:00	Yoonbok Lee	Miaofen Chen	Yeansu Kim	Ting-Yu Lee	Jaehyun Cho
10:00-10:30	Rest				
10:30-11:30	Masahiro Mine	Naoki Imai	Li Cai	Seidai Yasuda	Jia-Wei Kuo
11:30-14:00	Rest				
14:00-15:00	Yang Cao	Sho Yoshikawa	Yeongseong Jo	Ming-Hsuan Kang	Julie Tzu-Yueh Wang
15:00-15:30	Rest				
15:30-16:30	Haoyu Hu	Yukako Kezuka	Fu-Tsun Wei	Youngmin Lee	Daxin Xu

The following time table is Beijing time.

Taipei time is the same as Beijing time.

Tokyo time and Seoul time is one hour later than Beijing time.

<b>Day 1: Feb. 13 (Monday)</b>		
<b>Morning Session</b>		<b>Chair</b>
09:00-10:00	<b>Yoonbok Lee</b> <i>Value distribution of L-functions</i>	<b>Fei Xu</b>
10:00-10:30	<b>Rest</b>	
10:30-11:30	<b>Masahiro Mine</b> <i>On the denseness of the values of the Hurwitz zeta-function</i>	<b>Fei Xu</b>
<b>Afternoon Session</b>		
14:00-15:00	<b>Yang Cao</b> <i>Relative version of weak approximation for abelian schemes</i>	<b>Sunghan Bae</b>
15:00-15:30	<b>Rest</b>	
15:30-16:30	<b>Haoyu Hu</b> <i>Boundedness in étale cohomology</i>	<b>Sunghan Bae</b>

## Day 2: Feb. 14 (Tuesday)

Morning Session		Chair
09:00-10:00	<b>Miaofen Chen</b> <i>Harder-Narasimhan stratification in <math>p</math>-adic Hodge theory</i>	<b>Jilong Tong</b>
10:00-10:30	<b>Rest</b>	
10:30-11:30	<b>Naoki Imai</b> <i>Cohomology of moduli spaces of mixed characteristic local shtukas</i>	<b>Jilong Tong</b>
Afternoon Session		
14:00-15:00	<b>Sho Yoshikawa</b> <i>Modularity of elliptic curves over totally real quintic fields</i>	<b>Liang-Chung Hsia</b>
15:00-15:30	<b>Rest</b>	
15:30-16:30	<b>Yukako Kezuka</b> <i>Non-vanishing theorems for the Gross family of elliptic curves</i>	<b>Liang-Chung Hsia</b>

## Day 3: Feb. 15 (Wednesday)

Morning Session		Chair
09:00-10:00	<b>Yeansu Kim</b> <i>The generic unitary dual for odd <math>GSpin</math> groups and its applications</i>	<b>Shun Tang</b>
10:00-10:30	<b>Rest</b>	
10:30-11:30	<b>Li Cai</b> <i>Top-degree Ext-groups and relatively supercuspidal representations</i>	<b>Shun Tang</b>
Afternoon Session		
14:00-15:00	<b>Yeongseong Jo</b> <i>Rankin-Selberg integrals in positive characteristic and its connection to Langlands functoriality</i>	<b>Naoki Imai</b>
15:00-15:30	<b>Rest</b>	
15:30-16:30	<b>Fu-Tsun Wei</b> <i>Chowla-Selberg phenomenon over function fields</i>	<b>Naoki Imai</b>

## Day 4: Feb. 16 (Thursday)

<b>Morning Session</b>		<b>Chair</b>
09:00-10:00	<b>Ting-Yu Lee</b> <i>Oriented embedding functors and homogeneous spaces</i>	Yeansu Kim
10:00-10:30	<b>Rest</b>	
10:30-11:30	<b>Seidai Yasuda</b> <i>Local new forms and local L-factors for the general linear groups</i>	Yeansu Kim
<b>Afternoon Session</b>		
14:00-15:00	<b>Ming-Hsuan Kang</b> <i>On the zeta function of complexes of type <math>A_n</math>.</i>	Fu-Tsun Wei
15:00-15:30	<b>Rest</b>	
15:30-16:30	<b>Youngmin Lee</b> <i>Congruence properties for the number of partitions</i>	Fu-Tsun Wei

## Day 5: Feb. 17 (Friday)

<b>Morning Session</b>		<b>Chair</b>
09:00-10:00	<b>Jaehyun Cho</b> <i>On the analytic average of elliptic curves</i>	Yuichiro Tagauchi
10:00-10:30	<b>Rest</b>	
10:30-11:30	<b>Jia-Wei Kuo</b> <i>Cuspidal divisor class groups of the modular curve.</i>	Yuichiro Tagauchi
<b>Afternoon Session</b>		
14:00-15:00	<b>Julie Tzu-Yueh Wang</b> <i>Cases of Vojta's general abc conjecture for orbifold entire curves and applications</i>	Jing Yu
15:00-15:30	<b>Rest</b>	
15:30-16:30	<b>Daxin Xu</b> <i>Exponential sums, differential equations and geometric Langlands correspondence</i>	Jing Yu

## Abstracts

**Title:** Top-degree Ext-groups and relatively supercuspidal representations

**Speaker** Li Cai (Capital Normal University)

**Abstract:** Relatively supercuspidal (RSC) representations are the analogue object of supercuspidal representations in the relative Langlands program. In this talk, using top degree Ext-groups via the Schneider–Stuhler duality, we consider the classification of RSC representations in terms of cuspidal supports and in terms of Arthur parameters for the associated spherical varieties. We shall focus on the Flicker–Rallis case. Joint work with Yangyu Fan (Capital Normal University).

**Title:** Relative version of weak approximation for abelian schemes

**Speaker:** Yang Cao (University of Science and Technology of China)

Over number field, weak approximation for algebraic varieties is about the density of rational points in adelic points. In this talk, for an abelian scheme over a rational variety, instead of considering the weak approximation, we consider the relative closed subgroup

generated by rational points in adelic points of the total space and study its relationship with cohomological obstruction.

**Title:** Harder–Narasimhan stratification in  $p$ -adic Hodge theory

**Speaker:** Miaofen Chen (East China Normal University)

**Abstract:** we will talk about the construction of Harder–Narasimhan stratification on the  $B_{dR}^+$ -Grassmannian and study its basic geometric properties, which generalizes the work of Dat–Orlik–Rapoport, Cornut–Peche–Irissarry, Nguyen–Viehmann and Shen. This is a joint work in progress with Jilong Tong.

**Title:** On the analytic average of elliptic curves

**Speaker:** Jaehyun Cho (UNIST)

**Abstract:** We study the distribution of analytic ranks of elliptic curves.

For example,

1. We show that under GRH for elliptic curve L-functions, there are few elliptic curves with large analytic ranks.
2. We show that the average analytic rank of elliptic curves with prescribed torsion is bounded under GRH and some conditions. This talk is based on the joint works with Keunyoung Jeong.

**Title:** Boundedness in étale cohomology

**Speaker:** Haoyu Hu (Nanjing University)

**Abstract:** The estimation of Betti numbers of  $\ell$ -adic sheaves on positive characteristic varieties plays a crucial role in the study of exponential sums by using Arithmetic Geometry. In this talk, I will introduce a work on the bound of Betti numbers of étale sheave on smooth varieties of positive characteristic with bounded ranks and ramifications. This is a joint work in progress with Jean-Baptiste Teyssier.

**Title:** Cohomology of moduli spaces of mixed characteristic local shtukas

**Speaker:** Naoki Imai (University of Tokyo)

**Abstract:** Moduli spaces of mixed characteristic local shtukas are constructed by Scholze, and we can regard them as moduli spaces of modifications between two  $G$ -bundles over the Fargues-Fontaine curve. When two  $G$ -bundles are not semistable, it is already a non-trivial question how to define  $l$ -adic cohomology of the moduli space of modifications between the two  $G$ -bundles so that it gives smooth representations of  $p$ -adic groups. In this talk, we explain a definition of the  $l$ -adic cohomology of the moduli space, and how to relate cohomology of different moduli spaces.

**Title: Rankin–Selberg integrals in positive characteristic and its connection to Langlands functoriality**

**Speaker: Yeongseong Jo** (Ewha Womans University)

**Abstract:** The prominent Langlands functoriality conjecture predicts deep relationships among representations on different groups. One of the well-understood cases is a local functorial transfer of irreducible generic supercuspidal representations of  $\mathrm{SO}_{2r+1}(F)$  to irreducible supercuspidal ones of  $\mathrm{GL}_{2r}(F)$  over  $p$ -adic fields  $F$ . This was first pursued by Cogdell and Piatetski–Shapiro in 1990’s. This functorial lift is recently defined by Lomel’  $\{i\}$  over non-archimedean local fields  $F$  of positive characteristic. Although cases in characteristic zero  $F$  are settled by Jiang and Soudry, those in positive characteristic  $F$  is rarely studied. The purpose of this talk is to take one step further to investigate the transfer thoroughly. We consider the image of the map. Somewhat surprisingly, this is related to poles of local exterior square  $L$ -functions via integral representations due to Jacquet and Shalika. We then discuss whether the map is injective, which is in turn relevant to what is known as the local converse theorem for  $\mathrm{SO}_{2r+1}(F)$ .

**Title: On the zeta function of complexes of type  $A_n$ .**

**Speaker: Ming-Hsuan Kang** (Yang Ming Chiao Tung University)

Abstract: In this talk, we will explore the zeta functions of complexes arising from  $PGL(n+1)$ , which are high-dimensional analogs of Ihara zeta functions. We will demonstrate that these combinatorial geodesics counted by the zeta functions possess desirable geometric properties and are the natural generalization of geodesics on graphs. Besides, we will establish a connection between the zeta functions and the Langlands L-function of the complex. Finally, we will discuss how to extend our results to more general complexes of type  $A_n$ .

**Title: Non-vanishing theorems for the Gross family of elliptic curves**

**Speaker: Yukako Kezuka** (Institut de Mathématiques de Jussieu in Paris)

Abstract: The arithmetic of elliptic curves with complex multiplication has attracted many mathematicians, especially in the context of the Birch-Swinnerton-Dyer conjecture. Amongst these curves, Gross has introduced a particularly nice elliptic curve with complex multiplication. I will prove non-vanishing theorems for the central values of the complex L-series of a large class of quadratic twists of the Gross elliptic curve. In particular, we obtain the

finiteness of the Tate–Shafarevich group for these curves. The proof is based on Iwasawa theory at the prime  $2$ . This is joint work with Yong–Xiong Li.

**Title:** The generic unitary dual for odd *GSpin* groups and its applications

**Speaker:** Yeansu Kim (Chonnam National University)

**Abstract:** The unitary dual problem is to classify all unitarizable irreducible representations of  $G(F)$ , where  $G$  is a connected reductive group defined over a local field  $F$ . We introduce a generic case of the unitary dual problem in the case when  $G$  is an odd *GSpin* groups defined over  $F$  where  $F$  is a non-archimedean local field of any characteristic, i. e., generic unitary dual for odd *GSpin* groups. The variation of the project will be part of the joint work with L. Lomeli and H. Castillo.

**Title:** Cuspidal divisor class groups of the modular curve.

**Speaker:** Jia-Wei Kuo (Taiwan University)

**Abstract:** Let  $N = p^2M$  with  $p$  a prime and  $M$  a square free integer,  $\mathcal{C}(N)$  be the subgroup of the Jacobian variety  $J_0(N)$  generated by the equivalent classes of cuspidal divisors of degree  $0$ , and  $\mathcal{C}_{\mathbb{Q}}(N)$  be the subgroup of  $\mathcal{C}(N)(\mathbb{Q})$  generated by  $\mathbb{Q}$ -rational cuspidal divisors.

In this talk, we will show that the two groups  $\mathcal{C}(N)(\mathbb{Q})$  and  $\mathcal{C}_{\mathbb{Q}}(N)$  are equal by appropriate constructions of modular units. This is a joint work with Yifan Yang, Hwajong Yoo, and Myungjun Yu.

**Title:** Oriented embedding functors and homogeneous spaces

**Speaker:** Ting-Yu Lee (Taiwan University)

**Abstract:** Given a root datum and a reductive group scheme  $G$  over a locally noetherian scheme, we define an oriented embedding functor, which is representable, such that each point of the functor corresponds to an embedding of the root datum in  $G$ . In this talk, I will show that a homogeneous space of  $G$  whose geometric stabilizer is a maximal torus is in fact isomorphic to an embedding functor. This is a joint work with Philippe Gille

**Title:** Value distribution of L-functions

**Speaker:** Yoonbok Lee (Incheon National University)

**Abstract:** We introduce recent progress on the joint value distribution of L-functions.

**Title:** Congruence properties for the number of partitions

**Speaker:** Youngmin Lee (Korea Institute of Advance Study)

Abstract: For a positive integer  $n$ , let  $p(n)$  be the number of partitions of  $n$ . In 1960, Newman conjectured that for any integers  $M$  and  $r$  with  $0 \leq r \leq M - 1$ , there are infinitely many positive integers  $n$  such that  $p(n) \equiv r \pmod{M}$ . In this talk, we explain our results related to Newman's conjecture. Moreover, we introduce an analogue of Newman's conjecture for the Fourier coefficients of a weakly holomorphic modular form and its applications such as the congruence properties for the number of  $t$ -core partitions. This work is joint with Dohoon Choi.

**Title:** On the denseness of the values of the Hurwitz zeta-function

**Speaker:** Masahiro Mine (Sophia University)

Abstract: In the early 20th century, Bohr and Courant proved that the set of the values of the Riemann zeta-function  $\zeta(s)$  on the vertical line  $\operatorname{Re}(s) = \sigma$  is dense in  $\mathbb{C}$  with any fixed real number  $\frac{1}{2} < \sigma \leq 1$ . The Hurwitz zeta-function  $\zeta(s, \alpha)$  is a generalization of the Riemann zeta-function, and it is believed that the denseness theorem for  $\zeta(s)$  is generalized to  $\zeta(s, \alpha)$  for any parameter  $\alpha$ . The denseness of the values  $\zeta(s, \alpha)$  is known to be true when  $\alpha$  is rational or transcendental, but there still remains the case where  $\alpha$  is algebraic irrational. In this talk, I will present a weak denseness

result for  $\zeta(s, \alpha)$  with quadratic irrational parameter  $\alpha$ , which asserts that any complex number  $z_0$  can be approximated by the values  $\zeta(s, \alpha)$  for arbitrary quadratic irrational parameters  $\alpha$  but with finite exceptions depending on  $z_0$ , etc.

**Title:** Cases of Vojta's general abc conjecture for orbifold entire curves and applications

**Speaker:** Julie Tzu-Yueh Wang (Institute of Mathematics, Academia Sinica, Taiwan)

**Abstract:** We show a truncated second main theorem of level one (i.e. the complex version of Vojta's general abc conjecture) for analytic maps into  $\mathbb{P}^2$  intersecting the coordinate lines in sufficiently high multiplicities. In particular, the exceptional set can be described explicitly. As an application, we prove Campana's orbifold conjecture for finite ramified covers of  $\mathbb{P}^2$  with three components admitting sufficiently large multiplicities. This is a joint work with Ji Guo.

**Title:** Chowla-Selberg phenomenon over function fields

**Speaker:** Fu-Tsun Wei (Tsinghua University, Taiwan)

**Abstract:** In this talk, I will first determine the algebraic relations among various special gamma values over function fields.

The result is based on the intrinsic relations between gamma values in question and periods of CM dual  $t$ -motives, which are interpreted in terms of their “distributions”. This enables us to express every “abelian” CM period by a suitable product of special gamma values (up to an algebraic multiple), and derive a Chowla - Selberg-type formula in the function field case.

**Title: Exponential sums, differential equations and geometric Langlands correspondence**

**Speaker: Daxin Xu** (Chinese Academy of Sciences, Beijing)

**Abstract:** In 1970's, Dwork established a relationship between the Bessel differential equation and the Kloosterman sums. Such a relationship can be regarded as an instance of the geometric Langlands correspondence for  $GL_2$ . In this talk, I will first review some classical results on exponential sums and differential equations, and then discuss some recent progress on generalizations of Dwork's result from the perspective of geometric Langlands correspondence. It is based on joint works with Xinwen Zhu, and Kamgarpour-Yi.

**Title: Local new forms and local L-factors for the general linear groups**

**Speaker: Seidai Yasuda** (Hokkaido University)

Abstract: In this talk, I will explain the theory of local new forms for the general linear groups over nonarchimedean local fields of characteristic zero. This talk is based on the speaker's joint work with Hiraku Atobe and Satoshi Kondo.

**Title: Modularity of elliptic curves over totally real quintic fields**

**Speaker: Sho Yoshikawa** (Gakushuin University)

Abstract: The Shimura–Taniyama conjecture, which is now a theorem, states that all elliptic curves over the rationals are “modular”, in the sense that its Hasse–Weil L-function is the L-function associated to a modular form. There have also been many works on a direct generalization of the Shimura–Taniyama conjecture to totally real fields  $F$ , especially when  $[F:Q] = 2, 3$ , or 4. In this talk, I will explain a recent result on such a generalization when  $[F:Q] = 5$ ; more precisely, among all elliptic curves over all totally real quintic fields, essentially all but finitely many elliptic curves are modular. This is a joint work with Yasuhiro Ishitsuka and Tetsushi Ito.